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Where Have All The Sidebands Gone?

by

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ABSTRACT

We attempt to use a three-dimensional numerical model of flow in a rotating fluid annulus subject to a horizontal temperature gradient to reproduce the observations of Hide, Mason and Plumb (1977) concerning the general occurrence of sideband structures in the regular regime of flow. We are able to confirm that the phases of sidebands tend to be locked in the way predicted by Hide, Mason, and Plumb (1977) but show that the frame in which the mean frequencies of main wave and sidebands are the same is (apart from one doubtful case), not stationary with respect to the apparatus. The amplitudes of the sidebands either decline steadily or are smaller than in the experiments.

1. THE EXPERIMENTAL EVIDENCE OF SIDEBANDS

In a recent study of fully developed baroclinic waves in a rotating fluid annulus subject to a horizontal temperature gradient (see Fig 1 for geometry and notation) Hide, Mason and Plumb (1977) (hereafter referred to as HMP), described the typical structure of the regular waves, based on temperature measurements from a ring of thermocouples at the mid-level and mid-radius of the annulus. The measurements were conveniently analysed by Fourier decomposition; the dominant spectral components were found to be:

(a) The main wave, say wavenumber M itself.

(b) The harmonics, wavenumbers 2M, 3M etc., of the main wave. Because of the tendency of the flow to take the form of a jet stream separating relatively isothermal regions, the temperature profile at mid-level and mid-radius tended to take the form of a square-wave; consequently the odd harmonics were more pronounced.

(c) The "sidebands" of the main wave, ie, wavenumbers M-1, M+1, and sometimes, M-2, M+2, etc.

(d) The "long waves", wavenumbers 1 and possibly 2.

This spectral structure was found to be common to a very large number of experimental runs. Only when the flow was just beyond the transition from axisymmetric flow to regular waves were the sidebands sufficiently weak to be indistinguishable from experimental noise. Near the transition to irregular flow the sidebands increased in amplitude, sometimes to such an extent that the dominant wavenumber varied with time, and the simple picture presented above broke down. As the flow became irregular the spectrum took the form of a broad peak with several waves of comparable amplitude, and an "inertial subrange" at higher wavenumbers where the amplitudes declined steadily with increasing M.

However, in less extreme cases, the sidebands were always observed; the ratio of the sideband amplitude to the main wave amplitude was typically .1 to .5.

The experimental measurements of the spectral structures enabled the frequency, ω_m, of wavenumber m to be measured. The frequency of the harmonics was simply related to that of the main wave; i.e.

$$\omega_M = \tfrac{1}{2}\omega_{2M} = \tfrac{1}{3}\omega_{3M} \text{ etc} \qquad (1.1)$$

This simply implies that the main wave and its harmonics had the same phase speed, so that the wave preserved its shape as it propagated around the annulus. When the endwalls of the annulus were flat it was found that the mean frequencies of the sidebands were identical to that of the main wave:

$$\omega_M = \omega_{M+1} = \omega_{M-1} \qquad (1.2)$$

Such a relationship can be interpreted in terms of a modulation of the amplitude of the main wave with azimuth, the envelope of the modulation being stationary with respect to the apparatus. A more general form of the relationship (1.2) is

$$\omega_M + M\bar{\omega} = \omega_{M-1} + (M-1)\bar{\omega} = \omega_{M+1} + (M+1)\bar{\omega} \qquad (1.3)$$

where $\bar{\omega}$ is the frequency of the wave envelope.

There appears to be no reason why the apparatus frame should have any particular significance as far as the wave envelope is concerned, and so HMP suggested the relationship (1.2) must hold as a result of some forcing of the flow caused by small departures of the apparatus or the thermocouple array from axial symmetry. This hypothesis was supported by observations made when the endwalls of the annulus were made to slope in opposite senses, thereby introducing considerable dispersion into the system. The sideband structures were still observed but the frequencies of the waves now satisfied the relationship (1.3);

i.e. the wave envelope drifted with respect to the apparatus. By introducing a sufficiently tall isolated obstacle on one endwall, the envelope was again locked to the apparatus frame.

The equations (1.3) can be combined to eliminate ϖ, yielding

$$2\omega_M - \omega_{M+1} - \omega_{M-1} = 0 \qquad (1.4)$$

If this equation is integrated with respect to time, a relationship between the phases of the waves emerges:-

$$2\phi_M - \phi_{M-1} - \phi_{M+1} = \Phi \qquad (1.5)$$

where ϕ_m is the phase of wavenumber m and Φ is a constant. Such a relationship was indeed observed to hold in the experiments, when Φ was averaged over long periods; the value of Φ was found to be close to π. This result can be qualitatively interpreted by the general consideration of a quadratically interacting system. HMP discussed the interaction of the main wave (wavenumber M) and its sidebands (wavenumber $M \pm \delta M$) with the long wave (wavenumber δM). Such a system forms two triads close to resonance. It was shown that $\Phi = \pi$ results if there is no net forcing of the long wave, assuming that the long wave is neutrally stable.

The question arises as to whether the sideband interaction is a general property of baroclinic systems, and therefore of possible significance in meteorological and oceanographical contexts, or whether the sidebands were forced by some imperfection in the apparatus. The locking of the wave envelope in the apparatus frame in the case of flat endwalls certainly suggests that the apparatus must have some effect on the development of sidebands though it is difficult to see where imperfections of the magnitude apparently required could arise. The types of imperfection which may be considered include geometric non-axisymmetries (certainly

very small) and non-axisymmetries of the thermal field, which may not be negligible but are likely to be small. The thermocouple array is known to be less accurately circular and centred and may have some effect; experimental measurements using neutrally buoyant beads to determine the flow field would settle this point. It should be noted that an azimuthal modulation of the wave amplitude is known from visual observation of the annulus flow in the absence of any thermocouple array (see HMP for discussion of this point and references).

The use of a three dimensional numerical model of the annulus system provides a means of eliminating all non-axisymmetries of the geometry and the thermal field (at least to a level of one part in 10^{14}, the precision of our computer), and of eliminating the thermocouple array. It was decided to attempt to simulate flows typical of those described in HMP and to determine whether sidebands formed, and if so whether any relationship such as (1.5) was satisfied. The experiments described in this preliminary report indicate that the sidebands are weaker or even evanescent in the model, though the phase relationship (1.5) is satisfied when they are present. The possible reasons for this are discussed in section (5).

2. THE NUMERICAL MODEL

The details of the numerical model are extensively described in Farnell and Plumb (1975), Farnell and Plumb (1976) and Farnell (1977), and will not be repeated here, beyond an indication of the most important features.

The model solves a finite difference analogue of the Navier-Stokes equations for the motion of incompressible Boussinesq fluid in a rotating annular system. The difference scheme is based on the explicit "leap-frog" scheme for the advection terms suggested by Piacsek and

Williams (1970), designed to be of second order accuracy, and designed to minimize errors due to the finite differencing in the physically important integrals of motion. Such errors as remain all vanish in a steady state for flows with no sources or sinks of fluid at the boundaries. A Dufort-Frankel formulation is used to stabilise the diffusion terms.

The grid spacing is variable in both the radial and vertical directions in order to combine adequate resolution of the boundary layers with acceptable demands on computer storage; the degree of stretch is variable and is generally chosen so that there are 2 notional gridpoints in each boundary layer, and so that the resolution in the interior of the fluid is roughly isotropic.

A wide variety of boundary conditions are possible. There may be sources or sinks of fluid on the boundaries, which may be thermally insulating or conducting with a prescribed temperature distribution or heat flux. The surfaces may be either stress-free, or rigid. In order to facilitate comparison with laboratory measurements, the coefficients of viscosity, thermal conductivity and expansion are taken to be quadratic functions of temperature.

In the experiments to be described in this paper, we consider a simulation of the laboratory experiments in which the fluid is subjected to horizontal temperature gradient. Consequently, the endwalls are thermally insulating, the lower boundary being rigid and the upper free. The sidewalls are rigid and are maintained at a constant temperature. There are no sources or sinks of fluid. A resolution was chosen which uses our available resources to the full; the limiting factor was the size of high speed backing store (provided by the fixed-head disc on the IBM 360/195). We chose 16 points in the radial and vertical directions and 64 points in the azimuthal direction. An early attempt to run with

12 x 12 x 32 proved unsuccessful.

With such a configuration, the model requires 256 K bytes of main storage, and 3.44 M bytes of backing store (because it is not possible to eliminate dead space on the fixed head disc, we in fact occupy 4.7 M bytes of backing store) on the Met Office IBM 360/195 system. The computer time required is 2.3 seconds per timestep, though because of the time required to complete input/output operations, etc, the elapsed time is 9.6 seconds per timestep. This can prove to be the limiting factor in determining the amount of computation which can be achieved.

3. DESCRIPTION OF THE RUNS - THE WILLIAMS COMPARISON

A number of runs have been attempted, of which we shall describe three in this note. Their characteristics are noted in table I. Because the final state of the flow is not uniquely defined by the impressed parameters, the form of the initial fields used is significant. These invariably took the form of the steady axisymmetric flow with the given parameters, with a small perturbation applied to the temperature field at the four rings of points nearest to mid-level and mid-radius.

The first pair of runs were initially intended as a direct comparison with the integrations of Williams (1969, 1971), and were run using identical parameters. The main difference between Williams' model and ours is that we integrate the flow in the entire annulus, whereas Williams integrates in a sector of the annulus 72° wide, thereby excluding all waves but wavenumber 5 and its harmonics.

The first run we carried out was started with a wavenumber 5 perturbation, and evolved to a state in which only wavenumber 5 and its harmonics were present. The Williams run and ours are compared in fig (2). Figs (2a) and (2b) show the variation of zonally averaged temperature

across a meridional section in the Williams run and our run respectively. There is close agreement between the two fields, the only noticeable difference being in the +2.0K contour towards the upper boundary. Figs (2c) and (2d) plot the variation of the amplitude of wavenumber 5 across the azimuthal plane. Again the agreement is close. Notice especially the similarity of the position and shape of the main maximum and of the subsidiary maximum in the upper left hand corner. The major differences occur towards the upper right hand corner, towards which our .1K and .2K contours extend further than do those of Williams.

The agreement is remarkably good in view of the following observations:-

(a) We resolve boundary layers better than Williams' can.

(b) Williams has better resolution in the interior, particularly in the azimuthal direction (Williams resolves each wave by 33 x 33 x 37 points, whereas we resolve each wave by approximately 16 x 16 x 13 points).

(c) Although both solutions nominally pertain to the final steady form of the flow, in which the fully developed wave is progressing steadily around the annulus, our solution is certainly not quite steady *, although the changes with time are certainly slight. Williams says that his 1969 run (fields at 400 seconds after the start of the integration) and 1971 run (at 517 seconds) differ slightly, suggesting that his flow too is not quite steady. Since our two solutions were perturbed in a somewhat different way, it is not possible to define times at which the runs should be identical.

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* A particularly sensitive measure of the steadiness is given by the variation of Nusselt number at the inner and outer walls - these should be constant and equal for a completely steady wave.

In this run, the facility for studying departures from five-fold symmetry was not utilised; since no amplitude was initially present in waves other than wavenumber 5, they are unable to grow either by linear instability processes (i.e. baroclinic instability) or by non-linear interactions. The second run was therefore initiated to test the possibility that the William's flow might also possess significant energy in waves which were not harmonic of the basic wave. The same physical parameters were employed, but the initial perturbation consisted of a white mixture of all waves.

Very early in the integration, it became apparent that the dominant wavenumber was 4, rather than 5. Prominent sidebands, wave numbers 3 and 5, developed, together with the long wave, wavenumber 1. The variation of the amplitudes of these waves with time is shown in figure (3).

A number of reasons why wavenumber 4 rather than 5 seems to be preferred by our model for these parameters might be advanced, not all of which involve deficiencies in our model. Williams originally chose to run in a sector of $2\pi/5$ radians after a run with the same number of points (33 x 33 x 37 points) spread around 2π radius. This is a highly anisotropic, and, in the azimuthal direction, rather poor resolution. We know from our early experiments that an azimuthal resolution of 32 points is not always adequate. Furthermore, and perhaps more importantly, Williams is not able to resolve the boundary layers as well as we can; consequently the rates of heat transfer across the sidewall boundaries may not be calculated as accurately as they are in our model. Such an effect might be expected to lead to a modification of the dominant wavenumber, which bears a direct relationship to the heat flux across the system.

Referring again to fig (3), the sidebands, although prominent between 200 and 300 seconds after the beginning of the integration, begin to decline thereafter, and drop exponentially to time 800 seconds, when the integration was terminated. They show no indication of tending to any steady amplitude, and by the end of the integration are only about 1% of the amplitude of the main wave. The decay time is about 200 seconds. Although it is more erratic at first, wavenumber 1 eventually decays at the same rate as the sidebands. It seems reasonable to assume that the run is tending to a final state with just the main wave and its harmonics travelling steadily around the annulus.

Despite the steady decay of the sideband components, it is evident from an examination of the quantity Φ (see equation 1.5) that a non-linear sideband interaction of the kind described by HMP is occurring in our model. Fig (4) shows the variation of Φ with time. During the initial part of the integration, it is close to 0, rising to values close to π around time 150 seconds. Throughout the remainder of the integration, the phase locking holds to a considerable degree of accuracy, indicating that the system is adjusted to produce no net forcing of the long wave. Discussion of the case when $\Phi = 0$ requires detailed information concerning the relevant interaction coefficients, but we advance the suggestion that this may correspond to <u>maximum</u> net forcing of the long wave. This is plausible in a period when the waves are growing quickly to their maximum amplitude.

The frequencies of the main wave and sidebands are fairly constant after time 300 seconds, and can be used to calculate the frequency of the longwave envelope. This is not stationary with respect to the apparatus frame (in contrast with the experimental results) but drifts; the drift period is 193 s. This appears to confirm HMP's suspicion that the locking of the longwave envelope to the apparatus frame is a

spurious effect caused by departures of the apparatus from axial symmetry.

4. DESCRIPTION OF THE RUNS: LABORATORY COMPARISON:

The Williams comparison was not instigated to study sidebands, and so it may be no surprise that the sidebands decline in amplitude. Because the Williams system is rather small, viscous effects are more important than in the laboratory systems discussed in HMP and so the location of his run on the regime diagram is rather different from that of HMP's experiments. The variation of sideband structure with position on the regime diagram has not yet been studied closely and so the possibility remains that sidebands might only be present in larger (lower viscosity) systems. It should be noted in this context, however, that HMP remark that increasing the viscosity appears to increase the strength of the sidebands.

Our third run (see table 1) is an attempt to produce strong sidebands. The available experimental data was examined, and the case with strongest sidebands which was amenable to numerical simulation was selected. Fig (5) shows the time averaged spectrum for this laboratory experiment and fig (6) the typical variation of Φ with time. Both a longwave and sidebands are well developed in the spectrum, together with first and second harmonics of nearly equal amplitude. There is some evidence of sideband structure to the harmonics. The quantity Φ is always close to π, though with some high frequency fluctuations.

The numerical experiment was started with an initial perturbation consisting of wavenumber 5 together with a smaller amplitude "white" perturbation. This ensured that wavenumber 5 developed as the dominant wave while providing sufficient energy in the other waves to enable them to grow to finite amplitude and to participate in non-linear energy exchanges.

The integration has currently been taken to time 1200 seconds. It reached a form of steady state (as evinced by the constancy of various integrals of the motion and the equality of the Nusselt numbers for the inner and outer walls) at about time 700 seconds. Fig (7) shows the time averaged spectrum for the period 700-1200 seconds, which should be compared with the corresponding experimental spectrum shown in fig (5). The relative amplitudes of the wavenumbers 1, 5, 10 and 15 are not dissimilar, but the sidebands are relatively much weaker in the numerical simulation, the ratio of sideband to main wave amplitude being 0.13 as opposed to around 0.22 to 0.33 in the laboratory experiments. Both spectra show sidebands to the harmonics of the main wave, but their amplitudes are not really similar.

Fig (8) shows the variation of the amplitude with time of wavenumber 5 (solid line), 4 (broken solid line) and 6 (dashed line). By time 500 seconds all three waves have reached their final amplitudes. The variations in the amplitude of the main wave appear to be dying away by 1200 seconds. The fluctuations in the amplitudes of the sidebands are more pronounced and do not seem to be dying away; however there is no evidence for any systematic decline of the sideband amplitudes, at least on timescales less than 10^3 seconds.

The greater variability of the amplitudes in this run than in the previous run can presumably be attributed to the fact that the system is closer to the irregular transition, and to the smaller effect of viscosity. We are sure that our viscosity is still sufficient to prevent spurious effects as there was no tendency for energy to accumulate in the smallest scales of motion; indeed the higher wavenumber amplitudes fell off roughly as $m^{-3/2}$ (in accordance with the experimental observations reported by HMP).

The variation of the phase locking function, Φ, with time, shown in fig (9), was much less smooth than in the corresponding laboratory experiment (fig 6). Although there was some tendency for values to be around π, the scatter was very large, and there were periods (eg, 400-500 seconds, 720-750 seconds) when the phase locking evidently broke down totally. The most convincing example of the type of locking described by HMP occurred between 100 and 400 seconds, when the sideband amplitudes were much larger than they were later.
By the end of the integration, the phase locking is not very apparent.

The implied frequency of the longwave envelope is highly variable and, by the end of the integration, not at all well defined, as the variation of the phase speed of the sidebands became so erratic that a simple estimation of their frequencies was nearly impossible. However, three values of the envelope frequency emerged, corresponding to the periods 100-400 seconds 500-700 seconds, and beyond 700 seconds. In the early stages the period of the longwave envelope was fairly well defined, at around 250 seconds. At time 400 seconds (when the phase locking broke down) the dynamics of the system evidently changed markedly; the phase speed of both sidebands and the longwave changed considerably. Indeed the phase speed of wavenumber 4 changed sign so that the wave was propagating in the opposite direction to the main wave. As a result, the envelope period became exceedingly short, around 27 seconds. This type of behaviour persisted up to time 700 seconds when the phase locking again broke down. Thereafter the frequencies of the sidebands were difficult to estimate, but not very different from that of the main wave. Thus we have the surprising result that the wave envelope is apparently almost stationary in the apparatus frame during this period, in agreement with HMP's observation. Because the motion of the sidebands is so irregular, we believe this is not a very significant result.

The breakdown of the phase locking, such as was observed around time 450 seconds and 730 seconds, appears to be correlated with periods when the amplitudes of the sidebands were not widely different from those of neighbouring waves. Under such circumstances, the sideband interaction is presumably not notably stronger than other non-linear interactions and so the phase locking characteristic of sideband interaction disappears. This point is illustrated by fig(10). Fig(10a)shows the spectrum averaged over the period 685-695 seconds, when the phase locking began to break down. The sidebands are not a prominant feature of the plot. Fig(10b) is a similar plot, but for the period 810-820 seconds, when Φ was close to π . The sidebands are much more distinct in this case.

5. DISCUSSION

Two principal conclusions are drawn from this work at its present preliminary stage:

(a) When the phase locking relationships, equations (1.3, 1.5) hold, there is generally no tendency for the longwave envelope to be locked in the apparatus frame. Instead it moves around the apparatus, sometimes in a rather irregular fashion.

(b) All the evidence suggests that the numerical annulus system is not capable of sustaining sidebands for indefinitely long periods at the amplitudes observed in the corresponding laboratory experiments. Apart from this, the experimental and numerical spectra show a reasonable measure of agreement, especially when it is borne in mind that the experimental data used here were not obtained with numerical comparisons in mind. Some appreciable refinement of the thermocouple array and the heating of the sidewalls is now considered desirable for such work.

Furthermore, our experience with the Williams comparison runs gives us no doubt that HMP's conjecture, that it may be possible to obtain different spectra with the same dominant wavenumber and the same physical parameters, is substantially correct if the initial conditions are carefully chosen, and at least for moderate lengths of time.

The important problem raised by this work is the resolution of the major conflict between the experiments and the numerical simulations, namely, the absence or weakening of the sidebands. There are two extreme possibilities; either the sidebands are forced by some irregularity in the apparatus, or they are damped by some deficiency in the numerical scheme. The first possibility was discussed by HMP and it is difficult to conceive of a mechanism which could modulate the main wave when the modulation travels around the apparatus, (as was the case in the experiments with sloping endwalls), though it does seem likely that irregularities in the apparatus must be responsible for locking the longwave envelope to the apparatus frame when the endwalls are flat.

The possibility that deficiencies in the numerical scheme might inhibit the sideband instability demands close consideration. The decay timescale of the sidebands in the Williams run is much shorter than the viscous timescale for the wavelengths concerned. We do not introduce any artificial viscosity and our finite difference scheme introduces no numerical damping. Thus these "quasi-linear" mechanisms seem unlikely to have the observed effect.

It is well known that any finite difference scheme must introduce spurious retardation of disturbances in the flow, a retardation which becomes greater as the number of points resolving the wave decreases. It was therefore suggested that the numerical change in the phase speed

of the waves might remove the resonance involved in the sideband interaction and so remove the possibility of instability. We now consider this to be unlikely, but further experiments designed to test this contention are planned. First, it will be recalled that the sideband interaction involves only a near-resonance of the longwave with the dominant wave and its sidebands. Secondly, all the waves involved in the interaction are resolved by 10-15 points in our system, so that the degree of retardation is in any case likely to be small. Thirdly, it is the degree of artificial dispersion between the dominant wave and sidebands which is likely to be important in modifying the non-linear interaction; this must be very small. Taken together, these three points lead us to suppose that the sideband interaction should be relatively insensitive to the details of the numerical scheme.

Whatever the cause of the discrepency between the laboratory and numerical experiments, its resolution must have important implications either for the design and interpretation of laboratory experiments or for the design of numerical schemes intended to model highly non-linear systems (this must include numerical forecast models). We are continuing with numerical work and with the analysis of the current runs, and look forward to the results of laboratory experiments designed to map out the occurrence of sidebands on the regime diagram. Together, these investigations should shed more light on a perplexing problem.

We wish to thank Dr R Hide and Dr P J Mason for helpful discussions.

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TABLE 1: RUNS UNDERTAKEN

Geometrical Parameters			Physical Parameters			Initial Perturbation
a (cm)	b (cm)	d (cm)	(s^{-1})	(K)	Working Fluid	
2	5	3	0.8	5	"Theoretical Water"	Wavenumber 5
2	5	3	0.8	5	"Theoretical Water"	White
3.80	8.43	12.00	2.0	10	Water	Wavenumber 5 and White Continuum

properties of "theoretical water":

$$\rho = 0.9978\ (1 - 2.054 \times 10^{-4} \lfloor \overline{T} - 22 \rfloor)\ g\ cm^{-3}$$
$$\nu = 0.01008\ cm^2\ s^{-1}$$
$$K = 0.00142\ cm^2\ s^{-1}$$

properties of water:

$$\rho = 0.9978\ (1 - 2.27 \times 10^{-4} \lfloor \overline{T} - 22 \rfloor - 5 \times 10 \lfloor \overline{T} - 22 \rfloor^2)\ g\ cm^{-3}$$
$$\nu = 0.00963(1 - 0.023 \lfloor \overline{T} - 22 \rfloor)\ cm^2\ s^{-1}$$
$$K = 0.00142(1 + 0.005 \lfloor \overline{T} - 22 \rfloor)\ cm^2\ s^{-1}$$

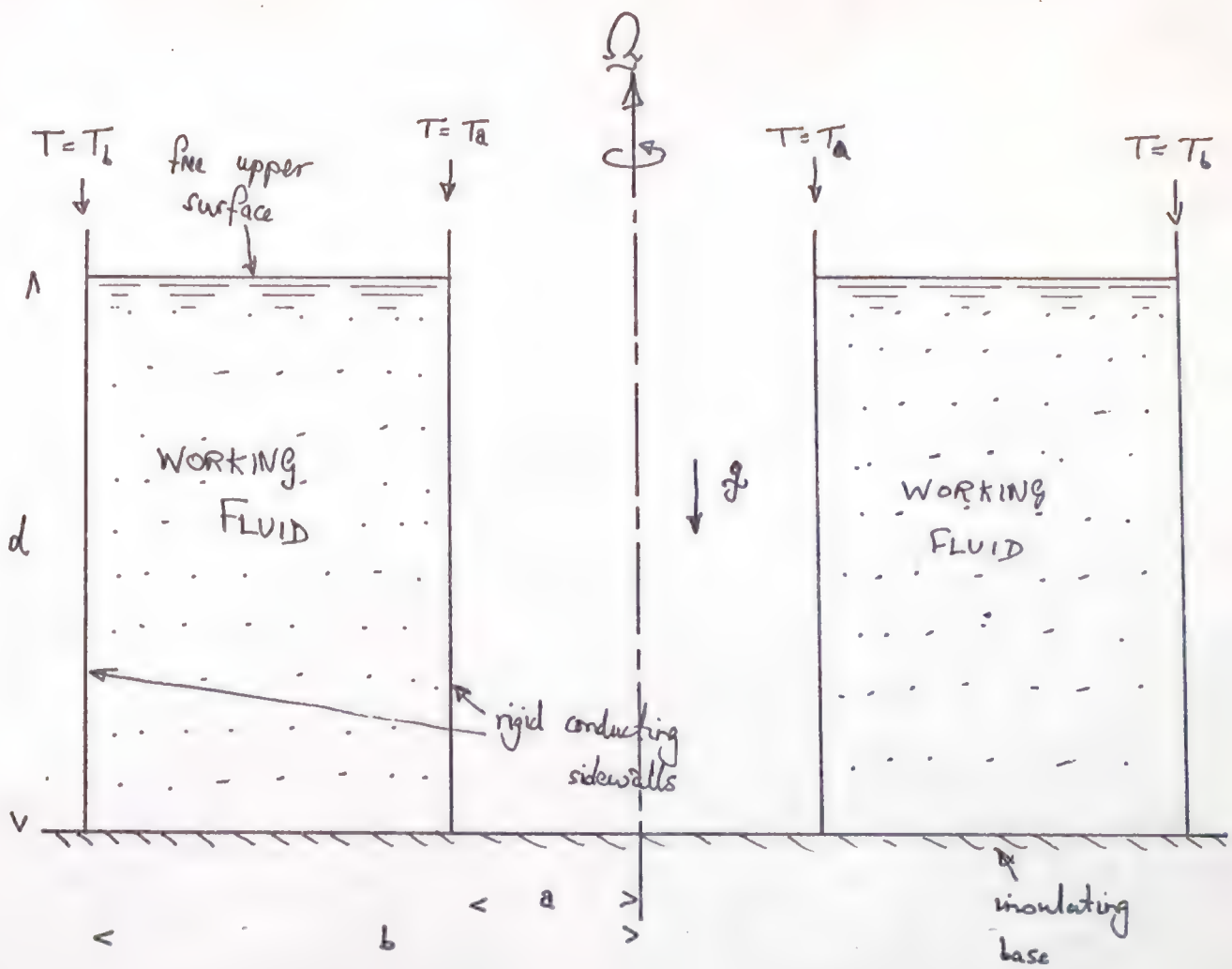
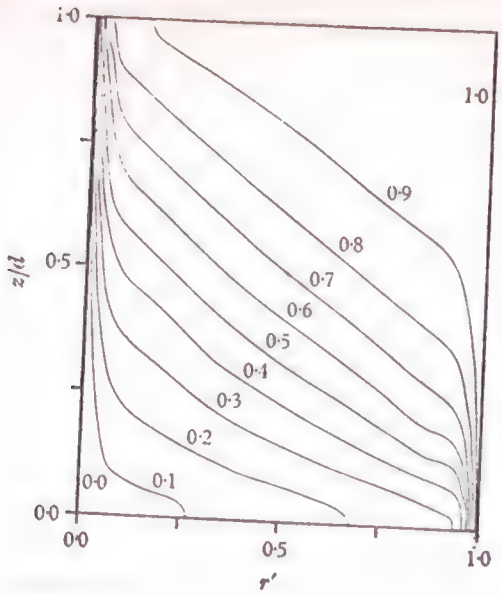
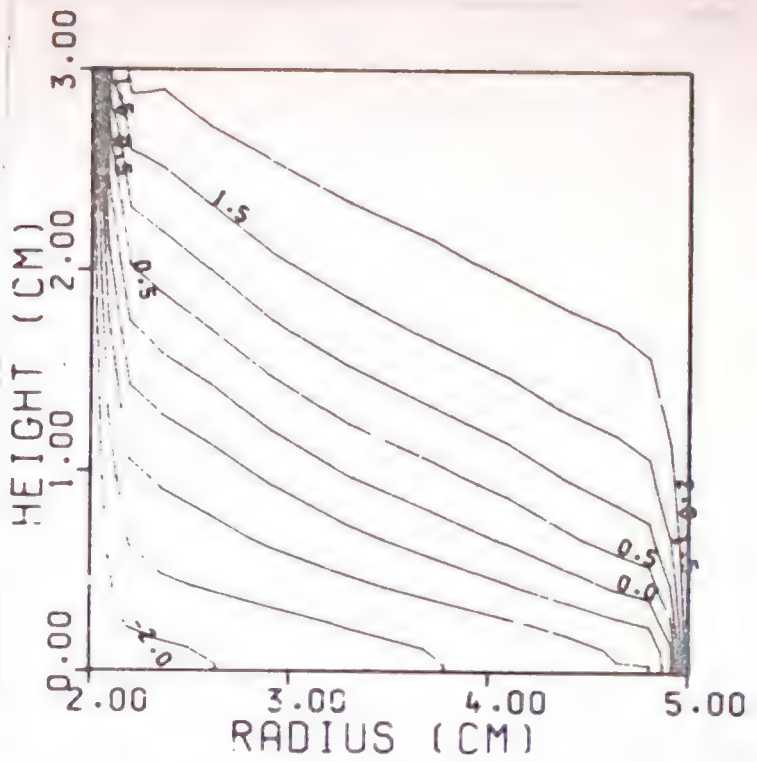


Fig (1): Meridional cross section through the configuration envisaged in the numerical experiments



(a)



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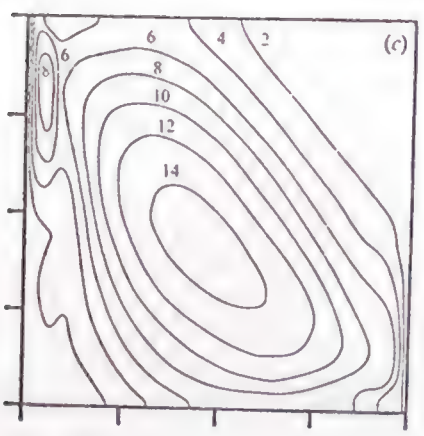

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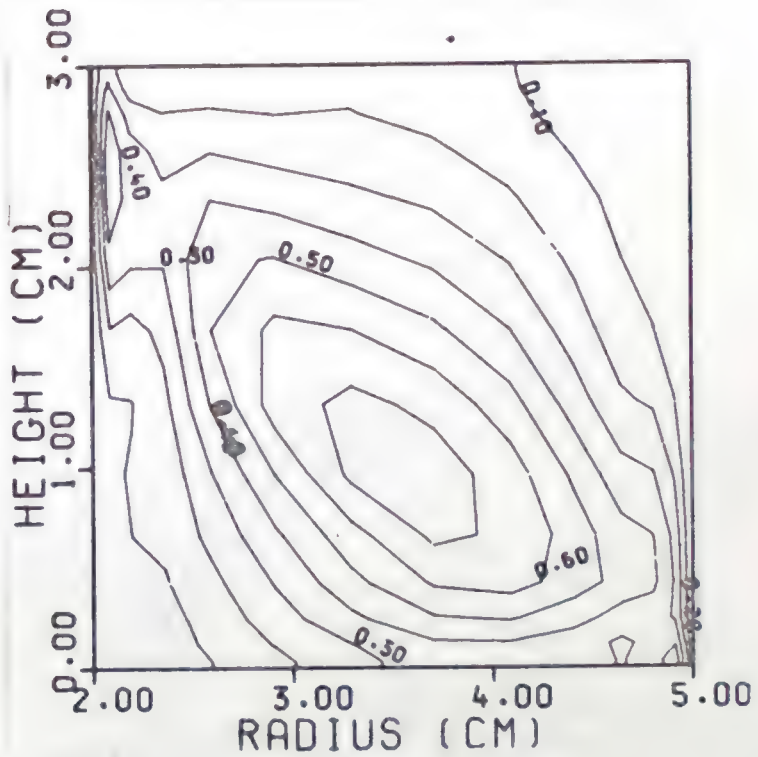


(d)

Fig (2): A comparison of Williams' integration of a wavenumber 5 flow. a) Zonally averaged temperature, Williams, b) Zonally averaged temperature, us, c) Variation of amplitude of wavenumber 5 with r and z, Williams, d) Variation of amplitude of wavenumber 5 with r and z, us. For details of labels on Williams' contours, see Williams (1969,1971). Our contours are labelled in K (relative to 22 K in the case of b)

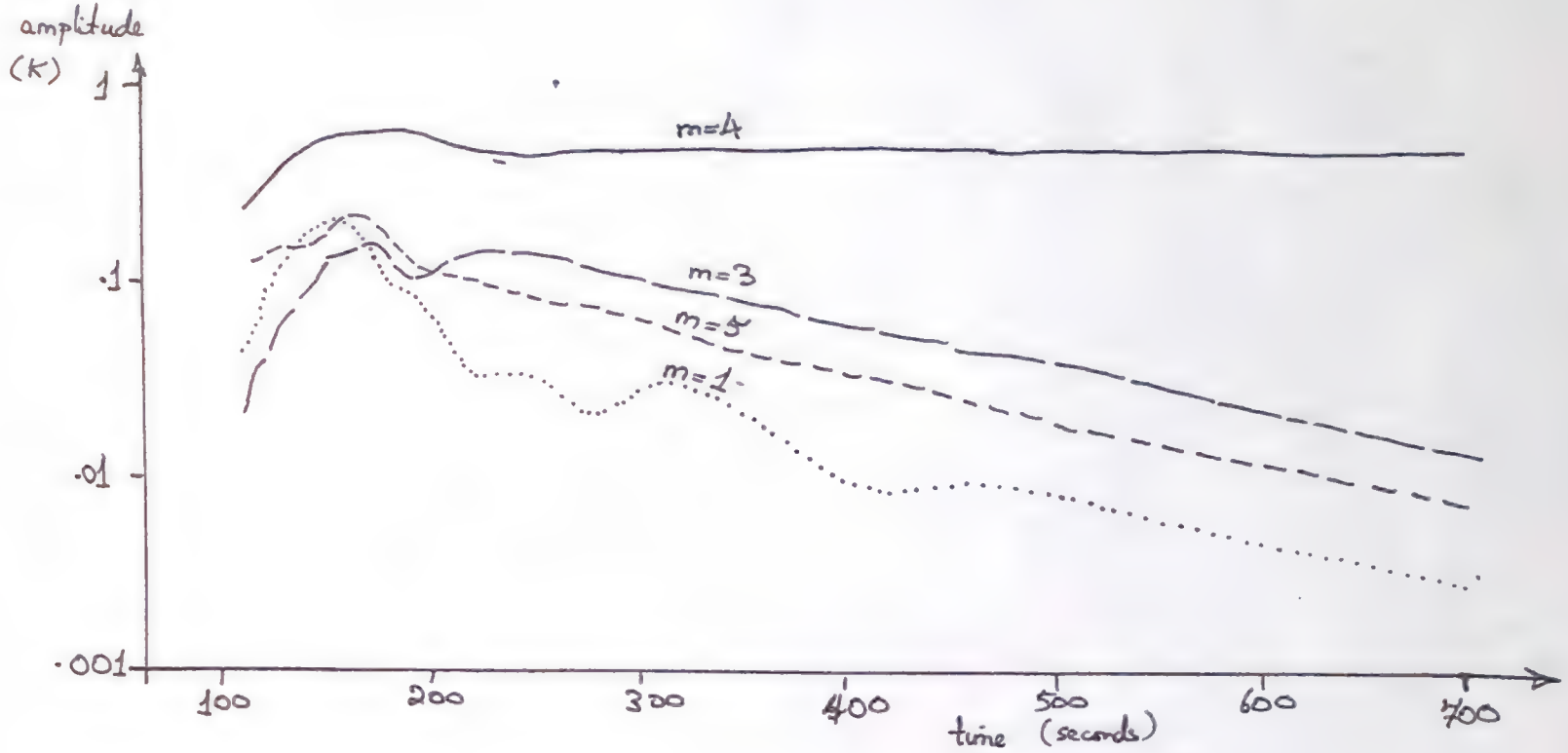


Fig (3): Variation of the amplitudes of wavenumbers 1,3,4 and 5 with time, Williams comparison run.

In this and subsequent figures, the analysis is of temperature at mid-level and mid-radius.

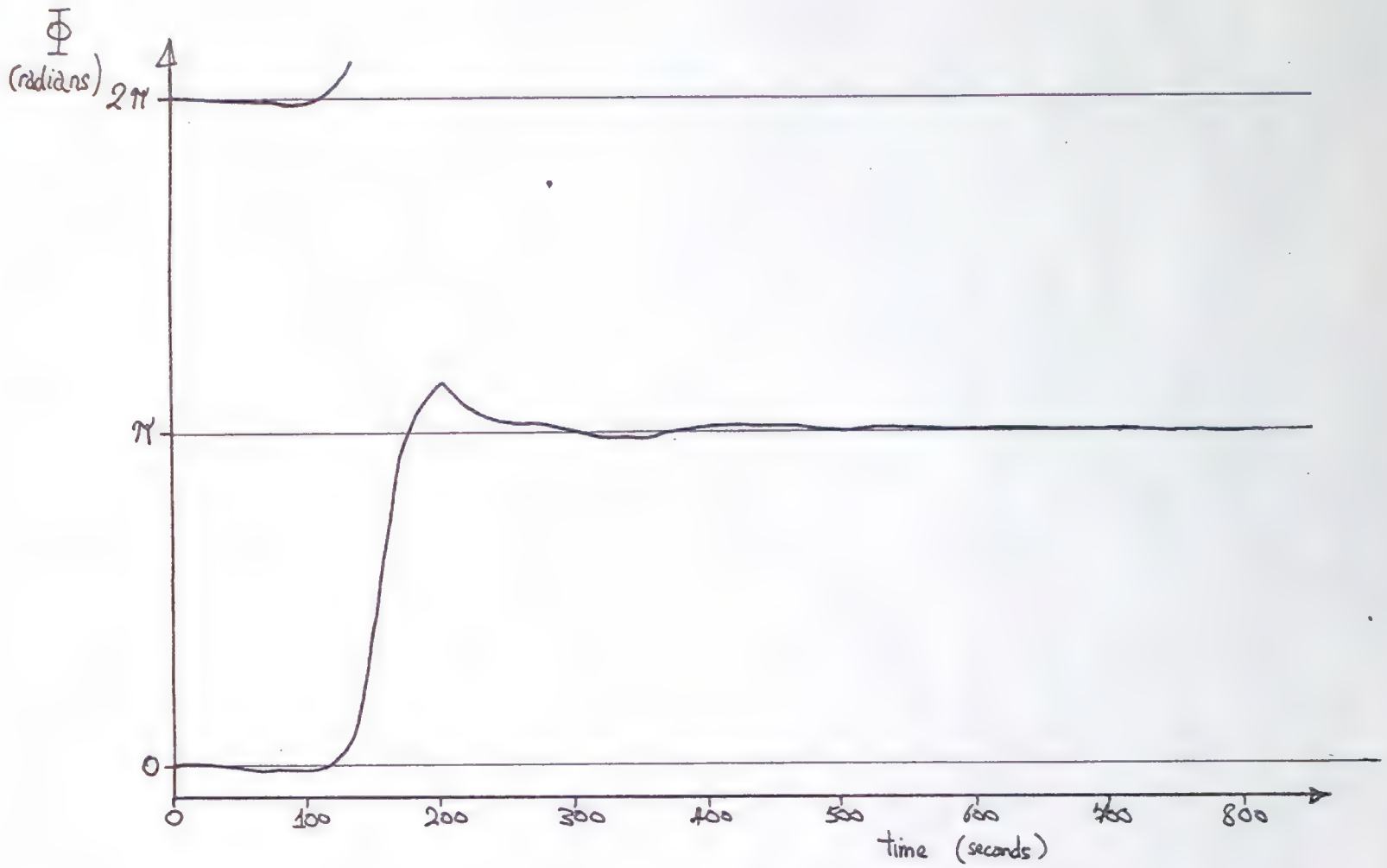


Fig (4): Variation of Φ with time, Williams comparison run.

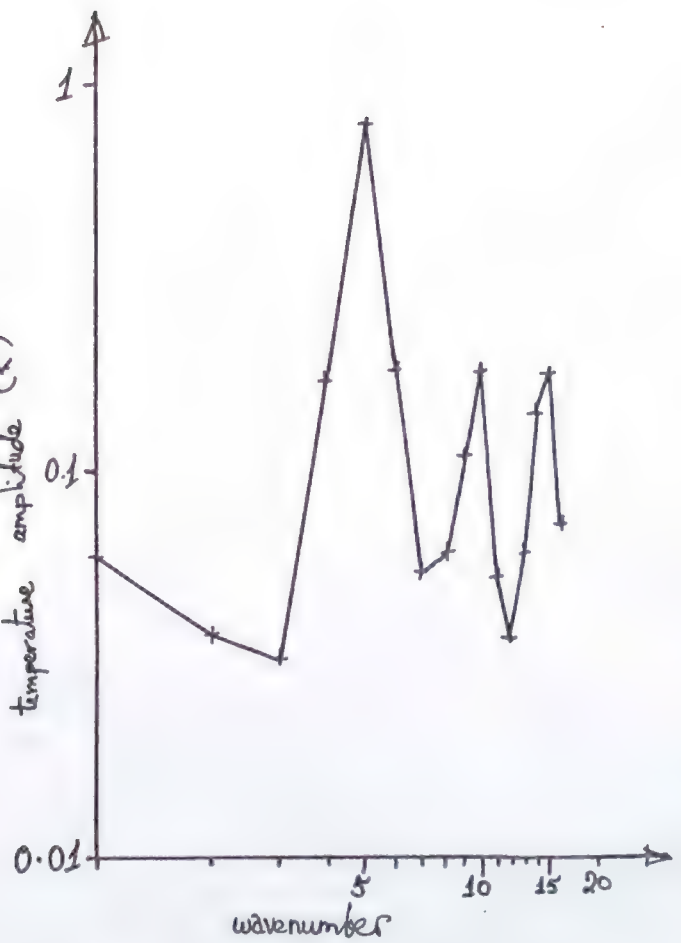


Fig (5): Time averaged spectrum for the experimental data selected for comparison. $\Omega = 2.56\ ^{c}s^{-1}$, $\Delta T = 9.9$ K. A second run with $\Omega = 1.9\ ^{c}s^{-1}$ which exhibited very similar characteristics was also taken into account in the comparison. Geometrical parameters are as in table 1.

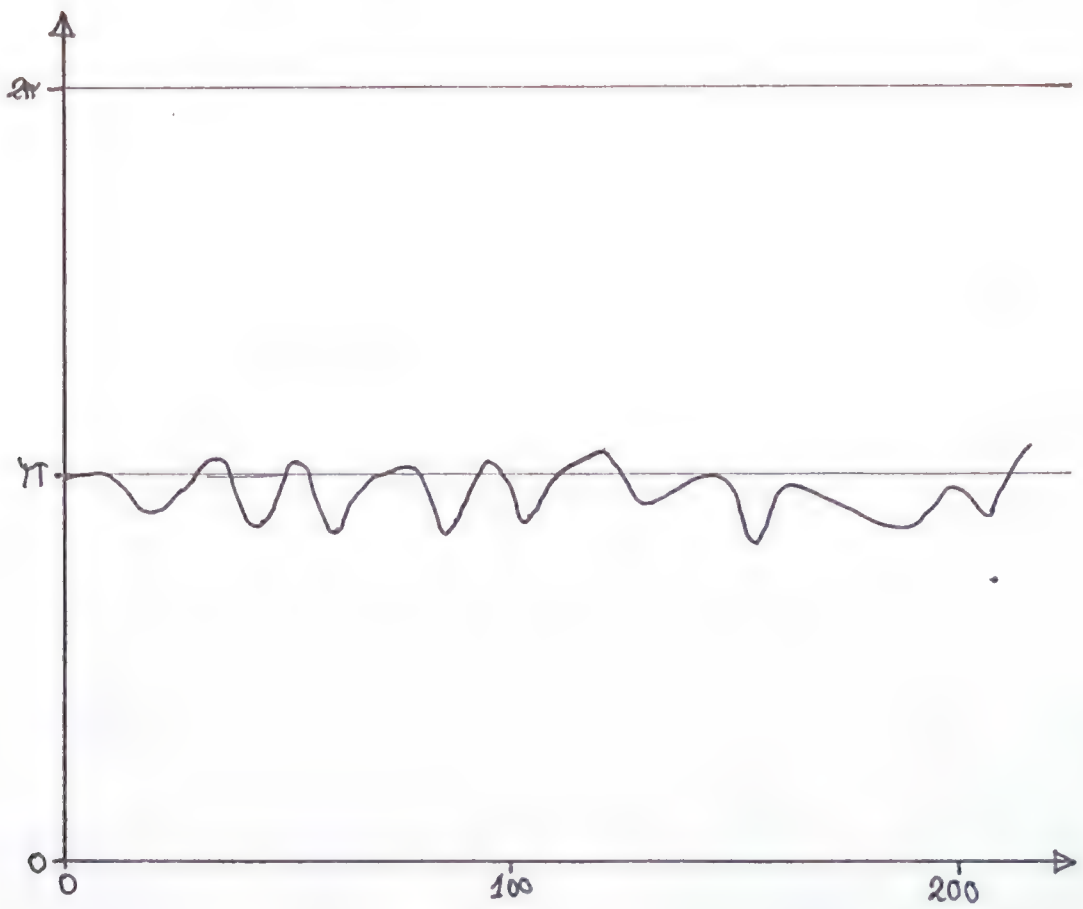


Fig (6): Variation of Φ with time for the laboratory experiment.

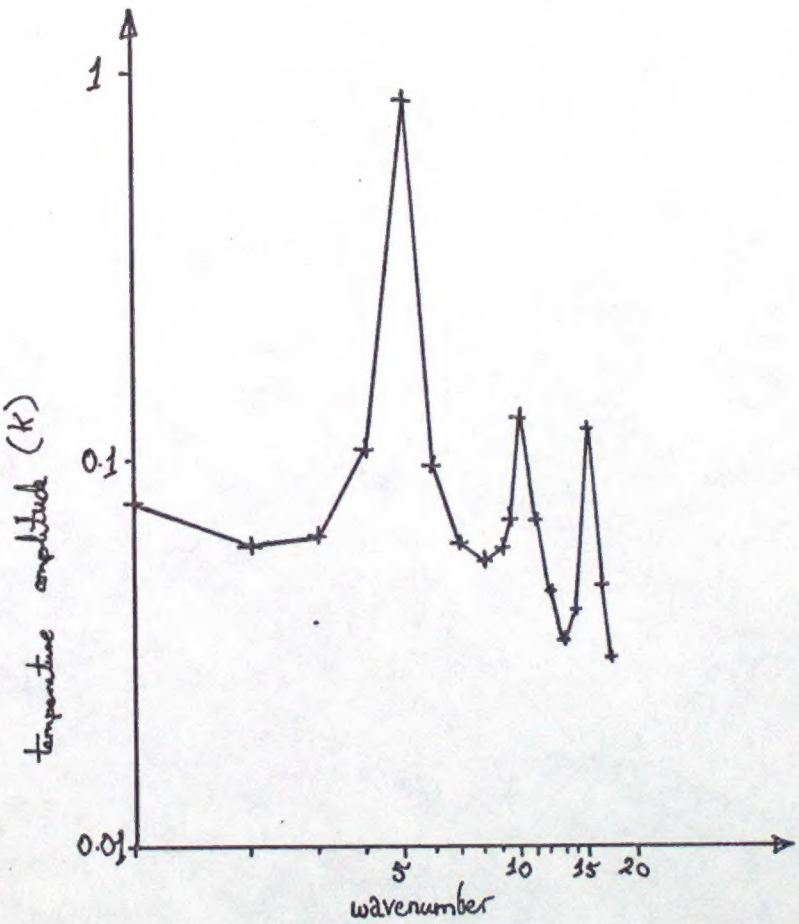


Fig (7): Time averaged spectrum (from time 700 to 1200 seconds) for the laboratory comparison run.

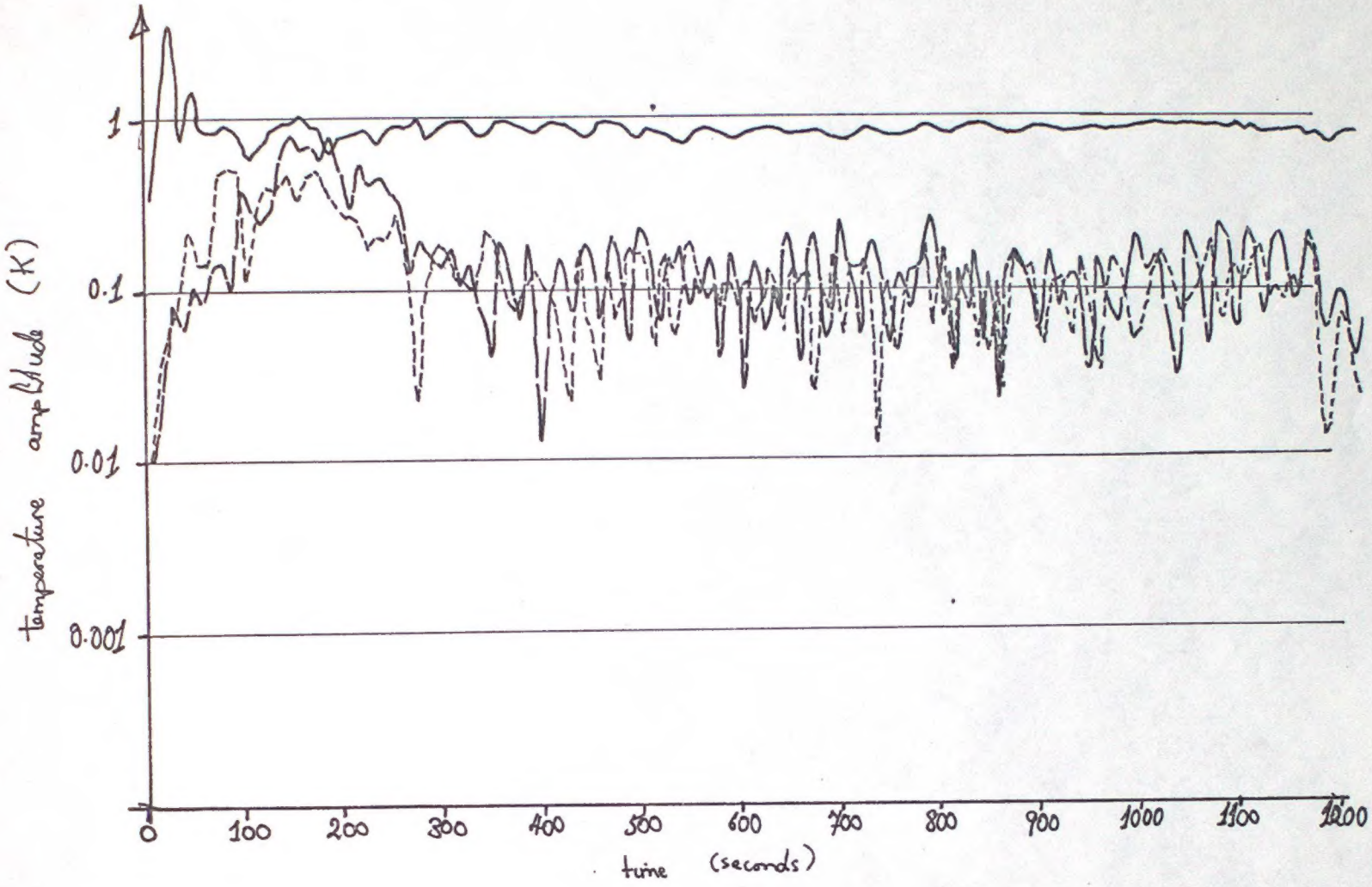


Fig (8): Variation of the amplitudes of wavenumbers 4 (——), 5 (——) and 6 (-----) with time, laboratory comparison run.

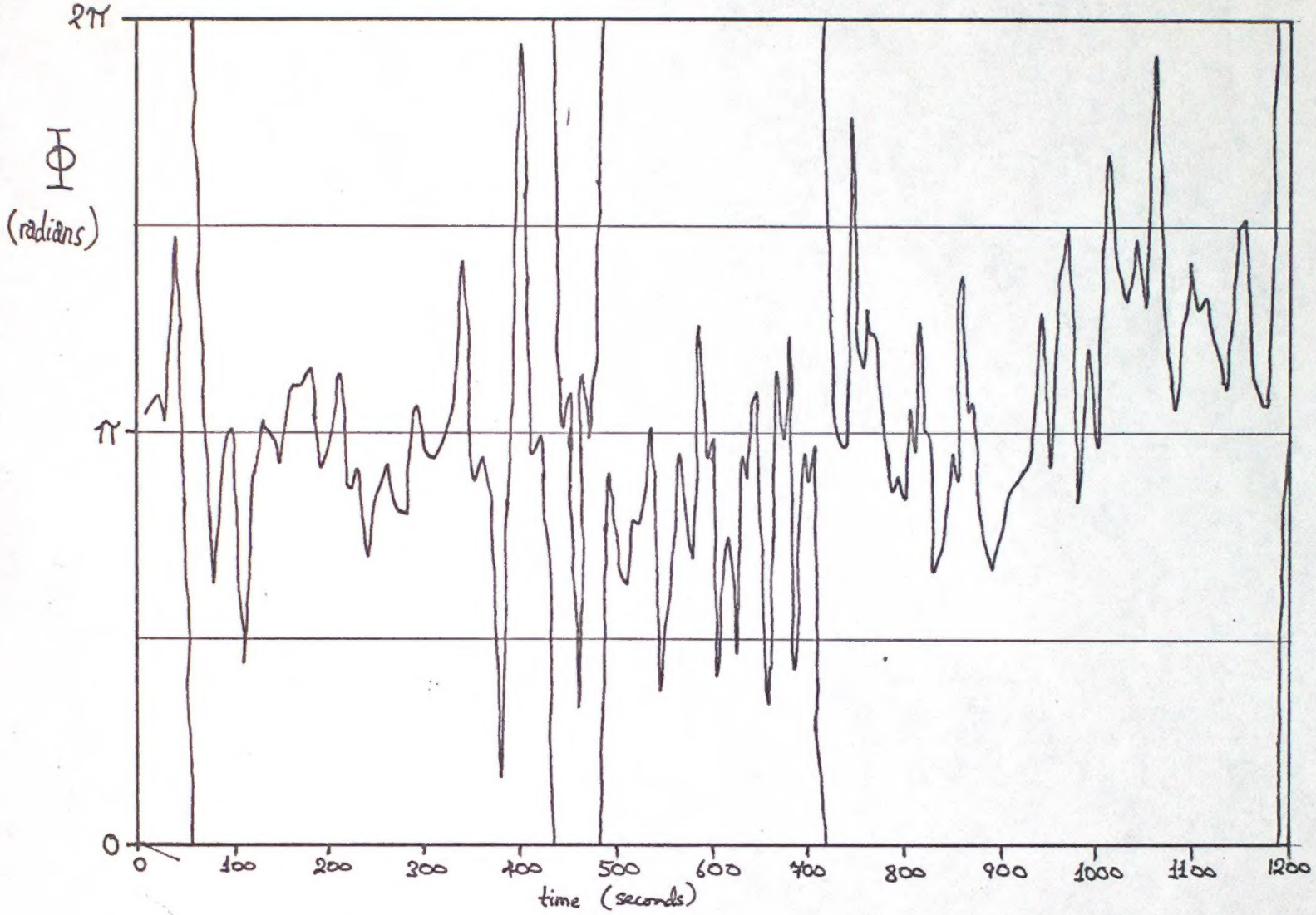
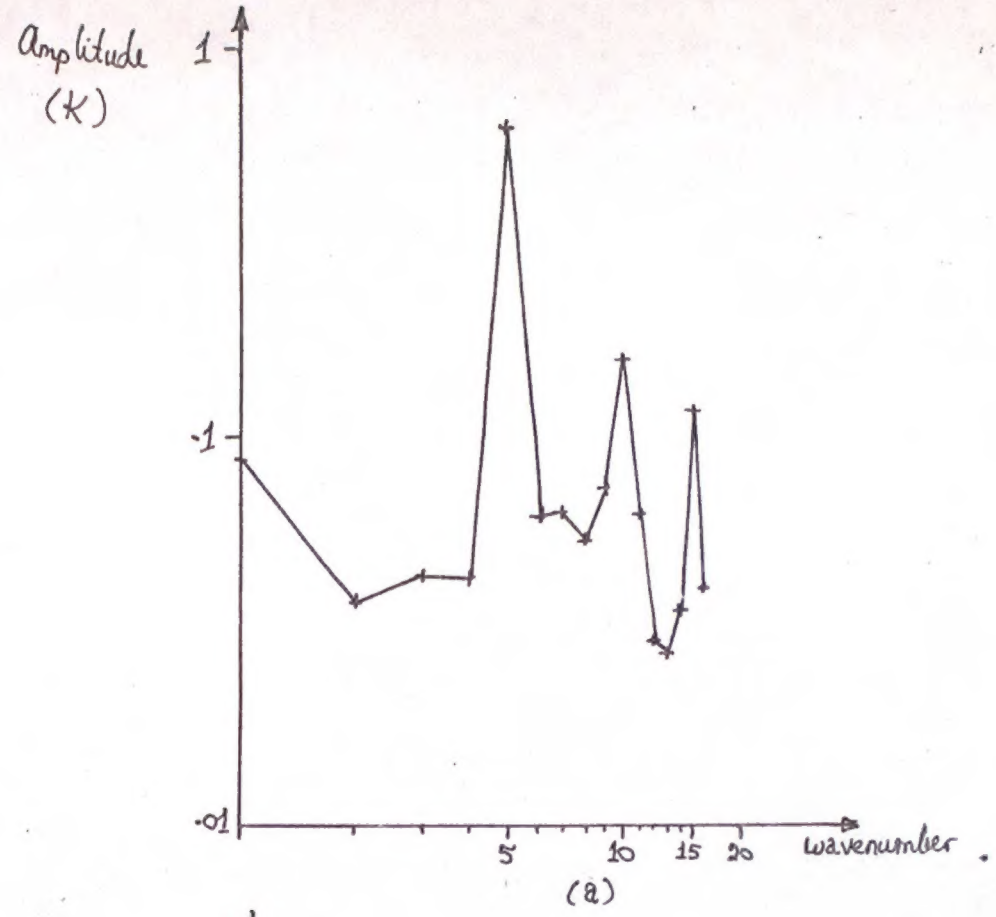


Fig (9): Variation of Φ with time, laboratory comparison run.



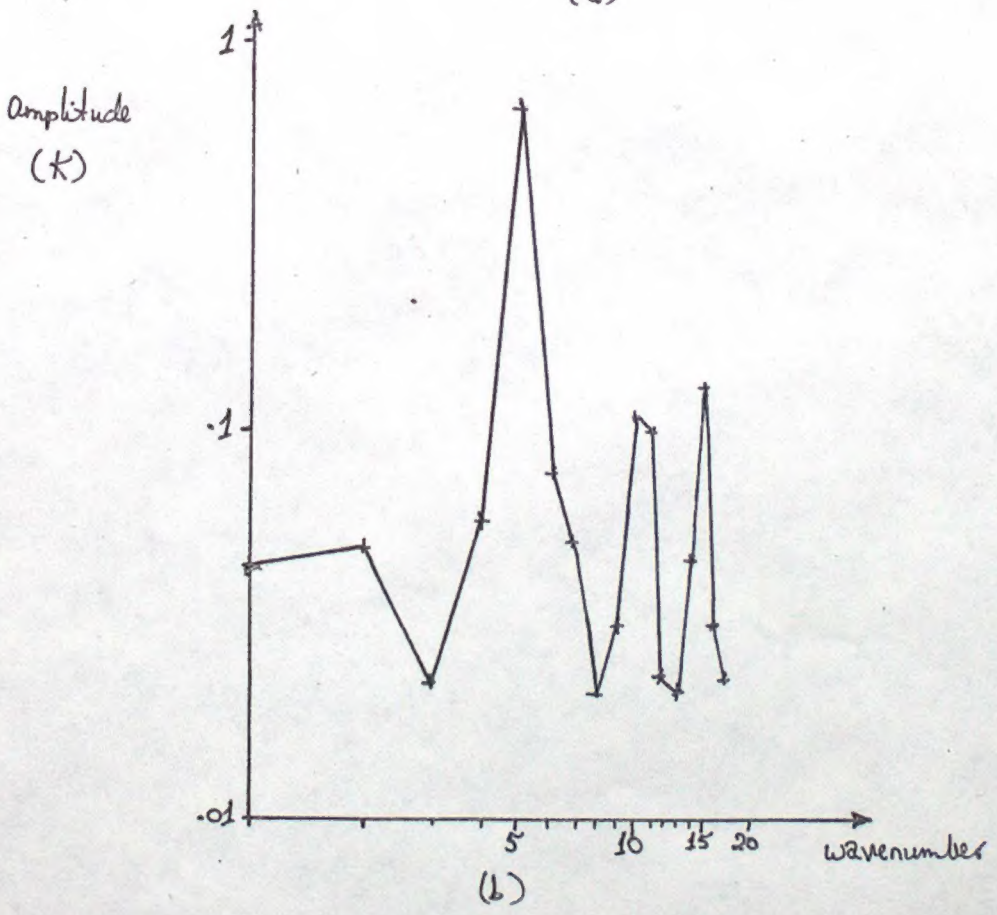


Fig (10): Time averaged spectra for the laboratory comparison run a) From 685 to 695 seconds, b) from 810 to 820 seconds.